Differentiation تفاضل

CH4 :- Derivative الإشتقاق

بيستر

4.1

The Derivative of a function

* The slope of line $= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Example:- Find the slope of line Passing through (1, 3) and (3, 4)

حل $S = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-3}{3-1} = \frac{1}{2}$

- Definition:- تعريف

The derivative of a function $y = f(x)$ at the Point x is the slope of tangent.

* Slope tangent at $x = a$ $= f(a)$

* Slope ميل
* tangent مماس

4.2

* Rules of Differentiation

① $\frac{d}{dx}(x^n) = n x^{n-1}$

② $\frac{d}{dx}(a x^n) = a (x^n)'$

① $f'(x)$
② $\frac{dy}{dx}$
③ $\frac{d}{dx}(f(x))$
④ y'

أوجد المشتقة الأولى

③ $\frac{d}{dx} a = 0$ $a =$ ثابت

④ $\frac{d}{dx}(x \cdot y) = (x \cdot y)' = x \cdot (y)' + y (x)'$

⑤ $\frac{d}{dx}\left(\frac{x}{y}\right) = \left(\frac{x}{y}\right)' = \frac{y(x)' - x(y)'}{y^2}$

⑥ $\frac{d}{dx}(h(x) \pm f(x)) = h'(x) \pm f'(x)$

7. $\left(\sqrt{f(x)}\right)' = \dfrac{f'(x)}{2\sqrt{f(x)}}$ مشتقة ما داخل الجذر على اثنين في نفس الجذر

8. $\dfrac{d}{dx}(x^n+x-9)^n = n(x^n+x-9)^{n-1}$ * (مشتقة ما داخل القوس)

ex) $f(x) = x^5$

① find $f'(x) = 5x^4$

② find the slope of tangent at $x=3$

$f'(x) = 5(3)^4 = 5(81) = 405$

بس يطلب أوجد ميل المماس عند نقطة أولاً جد المشتقة الأولى ثانياً عوض النقطة في المشتقة

* Questions

** find the $f'(x)$

① $f(x) = x^4$

$f'(x) = 4x^3$

(نتيجة للسعادة يكون النجاح)

② $f(x) = x^3 + 2x$

$f'(x) = 3x^2 + 2$

③ $f(x) = x^9 - 8$

$f'(x) = 9x^8$

④ $f(x) = 3x^9$

$f'(x) = 3(x^9)'$

$= 3(9x^8) = 27x^8$

⑤ $f(x) = 9x^3$

$f'(x) = 9(x^3)'$

$= 9(3x^2) = 27x^2$

⑥ $f(x) = 4x^3 - 2x + 5$

$f'(x) = 12x^2 - 2$

⑦ $f(x) = 3x^2 x^4$

$f'(x) = 3x^2(4x^3) + x^4(6x)$

$= 12x^5 + 6x^5 = 18x^5$

⑧ $f(x) = 3x^2 x^4$

$f(x) = 3x^6$

$f'(x) = 18x^5$

⑨ $f(x) = \frac{x^3 - 4x}{x^2}$ الطريقة الأولى

$f'(x) = \frac{x^2(3x^2 - 4) - (x^3 - 4x)(2x)}{(x^2)^2}$

$f'(x) = \frac{3x^4 - 4x^2 - (2x^4 - 8x^2)}{x^4}$

$= \frac{3x^4 - 4x^2 - 2x^4 + 8x^2}{x^4}$

$= \frac{x^4 + 4x^2}{x^4} = \frac{x^4}{x^4} + \frac{4x^2}{x^4}$

$= 1 + \frac{4}{x^2}$

⑨ $f(x) = \frac{x^3 - 4x}{x^2}$ الطريقة الثانية

$f(x) = \frac{x^3}{x^2} - \frac{4x}{x^2}$

$f(x) = x - \frac{4}{x}$

$f(x) = x - 4x^{-1}$

$f'(x) = 1 + 4x^{-2}$

$= 1 + \frac{4}{x^2}$

ملاحظات :

① الأسس في حالة الضرب تجمع

$x^5 \cdot x^1 = x^6$

② الأسس في حالة الرفع لقوى تضرب

$(x^5)^2 = x^{5\times2} = x^{10}$

③ الأسس في حالة القسمة تطرح

$\frac{x^5}{x^2} = x^{5-2} = x^3$

④ في حالة تشابه الأس بالجمع أو الطرح نجمع الرقم أو نطرحه

$2x^4 + 5x^4 = 7x^4$

$-2x^4 + 5x^4 = 3x^4$

(الغموض هو أجمل ما يمكن أن نشعر به إنه مصدر كل الفنون والعلوم الحقيقية)

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(10) $f(x) = \frac{4}{x}$

$f(x) = 4x^{-1}$

$f'(x) = -4x^{-2} = \frac{-4}{x^2}$

(11) $y = \sqrt{x^3}$ الطريقة الأولى

$y' = \frac{3x^2}{2\sqrt{x^3}}$ القيمة نفس

(11) $y = \sqrt{x^3}$ الطريقة الثانية

$y = (x^3)^{\frac{1}{2}} = x^{\frac{3}{2}}$

$y' = \frac{3}{2}x^{\frac{1}{2}} = \frac{3\sqrt{x}}{2}$

(12) $f(x) = x\sqrt{x}$

$f(x) = x^1 x^{\frac{1}{2}} = x^{\frac{3}{2}}$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}}$

(13) $y = \frac{\cancel{x}\sqrt{x^3}}{x^{\cancel{3}\,2}} = \frac{\sqrt{x^3}}{x^2}$

$y = \sqrt{x^3} \cdot x^{-2} = x^{\frac{3}{2}} \cdot x^{-2}$

$y = x^{\frac{3}{2}-2} = x^{\frac{-1}{2}}$

$y' = \frac{-1}{2}x^{\frac{-3}{2}} = \frac{-1}{2\sqrt{x^3}}$

(14) $f(x) = 99$

$f'(x) = 0$

معلومة

✗ $\sqrt{x} = x^{\frac{1}{2}}$

✗ الجذر $= x^{\frac{\text{داخل}}{\text{خارج}}}$

✗ $\sqrt[2]{x^{3}}$ (خارج ②، داخل ③) $= x^{\frac{3}{2}}$

✗ معلومة

$\frac{1}{x^2} = x^{-2}$

$x^{-5} = \frac{1}{x^5}$

Q) find gradient at $x=4$

$f(x)=4x^3$

gradient = (المشتقة)

الحل

$f'(x)=12x^2$

$f'(4)=12(4)^2$

$=12*16=192$

حل مثل هالسؤال
أولاً جد المشتقة الأولى
ومن ثم عوض القيمة في المشتقة

كل الشرور مصدرها الجهل
وكل الخير مصدرها المعرفة ☺

Q) find the $f'(x)$

بيكسار

① $f(x)=x^4-\sqrt{x}-\frac{1}{x^2}+10$

$f(x)=x^4-x^{\frac{1}{2}}-x^{-2}+10$

$f'(x)=4x^3-\frac{1}{2}x^{\frac{-1}{2}}-(-2x^{-3})$

كن شامخاً في تواضعك
ومتواضعاً في شموخك

$=4x^3-\frac{1}{2\sqrt{x}}+\frac{2}{x^3}$

② $y=x^2(3x^4+3x)$ العلاقة الأولى

$y'=x^2(12x^3+3)+(3x^4+3x)(2x)$

$y'=12x^5+3x^2+6x^5+6x^2$

$=18x^5+9x^2$

② $y=x^2(3x^4+3x)$

$y=3x^6+3x^3$

$y'=18x^5+9x^2$

* معلومة

① الإشارات في حالة الجمع

إشارتان موجبتان نضع موجب ونجمع

$+10+5=+15$

② الإشارات في حالة الجمع

إشارتان مختلفتان نضع إشارة الأكبر ونطرح

$+10-7=+3$

③ الإشارات في حالة الطرح

إشارتان متشابهتان نضع إشارة سالب ونجمع

$-10-7=-17$

④ الإشارات في حالة الضرب (القسمة)

- عند الضرب أو القسمة

* إشارتان متشابهتان نضع موجب ونضرب (نقسم)

$-10 * -2 = +20$

$\frac{-10}{-2} = +5$

* إشارتان مختلفتان نضع سالب ونضرب (نقسم)

$-10 * 2 = -20$

$\frac{-10}{2} = -5$

⑦

Q) If $y = x^{\frac{2}{3}}$ and the gradient $\frac{dy}{dx} = \frac{1}{3}$ find the Point x

الحل $y = x^{\frac{2}{3}}$

$$\frac{dx}{dy} = \frac{2}{3} x^{\frac{-1}{3}} = \frac{2}{3\sqrt[3]{x}}$$

$$\frac{2}{3\sqrt[3]{x}} = \frac{1}{3}$$ ¥ ضرب تبادلي (تقاطعي)

$$6 = 3\sqrt[3]{x}$$ ¥ قسمة 3 لكلا الطرفين

$$2 = \sqrt[3]{x}$$ ¥ تكعيب الطرفين

$$(2)^3 = (\sqrt[3]{x})^3$$

$$\boxed{x = 8}$$

Q) find the y'

① $y = (5x+1)(x^2+2x+3)$ find y' and $y'(2)$

$$y' = (5x+1)(2x+2) + (x^2+2x+3)(5)$$

$$= 10x^2 + 10x + 2x + 2 + 5x^2 + 10x + 15$$

$$= 15x^2 + 22x + 17$$

$$y'(2) = 15(2)^2 + 22(2) + 17 = 121$$

② $f(x) = \frac{x^2+3}{2x-1}$

$$f'(x) = \frac{(2x-1)(2x) - (2)(x^2+3)}{(2x-1)^2}$$

$$= \frac{4x^2 - 2x - 2x^2 - 6}{4x^2 - 4x + 1} = \frac{2x^2 - 2x - 6}{4x^2 - 4x + 1}$$

¥ المفكوك التربيعي

① $(x-y)^2 = (x^2 - 2xy + y^2)$

② $(x+y)^2 = (x^2 + 2xy + y^2)$

Q) find the $f'(x)$

① $f(x) = (2x^2 - 4x + 8)^3$

$f'(x) = 3(2x^2 - 4x + 8)^2 * (4x - 4)$

② $y = \frac{1}{(x^3 + 2x)^2} = (x^3 + 2x)^{-2}$

$y' = -2(x^3 + 2x)^{-3} * (3x^2 + 2)$

Q) Differentiate

① $y = x^5$

$y' = 5x^4$

Q) find the second derivative for $f(x) = x^5 + 3x^2$

(المشتقة الثانية)

الحل

$f'(x) = 5x^4 + 6x$

$f''(x) = 20x^3 + 6$

المشتقة الثانية $\frac{d^2y}{dx^2}$

** إذا لم تقاوم من أجل شيء تحبه
فلا تبك إذا خسرته

4.7 Marginal functions

① Marginal revenue function $= MR = \frac{d(TR)}{dQ}$

$(TR) = P.Q$

② Marginal cost function $= MC = \frac{d(TC)}{dQ}$

$(TC) = FC + VC(Q)$

$TC = Q\,AC$

③ $MPL = \frac{dQ}{dL}$ مشتقة مع

MPL → Marginal Product of Labour

④ $APL = \frac{Q}{L}$ بدون مشتقة

APL → Average Product of Labour

Q) let the demand function $= 60 - Q_d$, find

① Total revenue when $Q = 50$

$TR = P.Q$

$= (60 - Q)(Q) = (60Q - Q^2)$

$= 60Q - Q^2 = 60(50) - (50)^2 = 3000 - 2500 = 500$

② Total revenue when $Q = 51$

$TR = 60Q - Q^2$

$TR\big|_{Q=51} = 60(51) - (51)^2 = 3060 - 2601 = 459$

③ Marginal revenue when $Q = 50$

$TR = 60Q - Q^2$

$MR = \frac{d}{dQ}(TR)$

$= \frac{d}{dQ}(60Q - Q^2) = 60 - 2Q$

$MR\big|_{Q=50} \rightarrow 60 - 2(50) = 60 - 100 = -40$

Q) If the demand function is $P = 120 - 3Q$ find Marginal revenue at $Q = 10$

Sol

$TR = P \cdot Q$

$= (120 - 3Q)\,Q$

$= 120Q - 3Q^2$

$MR = \frac{d}{dQ}(TR)$

$= \frac{d}{dQ}(120Q - 3Q^2)$

$MR = 120 - 6Q$

$MR\big|_{Q=10} \longrightarrow 120 - 60 = 60$

Q) If $TR = 1000Q - 4Q^2$, find the Marginal revenue

Sol

$MR = \frac{d}{dQ}(TR)$

$= \frac{d}{dQ}(1000Q - 4Q^2)$

$MR = 1000 - 8Q$

Q) The $C = \frac{100}{Q} + 2$, find MC

Sol

$MC = \frac{d}{dQ}(TC)$

$= \frac{d}{dQ}\left(\frac{100}{Q} + 2\right)$

$= \frac{-100}{Q^2}$

Q) If the Average Cost function of a good is $AC = 2Q + 6 + \frac{13}{Q}$ find MC

حل

$TC = AC(Q)$

$TC = (2Q + 6 + \frac{13}{Q})\,Q$

$= 2Q^2 + 6Q + 13$

الحكيم يتفكر في تدابير الله

$MC = \frac{d}{dQ}(TC)$

$= 4Q + 6$

* Marginal Product of labour $= MPL = \frac{dQ}{dL}$

Example:-) If the Production function is $Q = 300\sqrt{L} - 4L$ Calculate MPL when $L = 100$

حل

Q → out Put
L → the size of workforce

$Q = 300(L^{\frac{1}{2}}) - 4L$

$MPL = \frac{1}{2}(L)^{\frac{-1}{2}}(300) - 4$

$= \frac{150}{\sqrt{L}} - 4$

$MPL|_{L=100} \rightarrow \frac{150}{\sqrt{100}} - 4 = \frac{150}{10} - 4 = 15 - 4 = 11$

استهلاك ادخار

* Consumption and saving

- we denote Consumption by C and saving by S

If the National Income is Y, then

$$Y = C + S$$

Now, MPC = Marginal Propensity to consume (ميل حدي الاستهلاك)

MPS = Marginal Propensity to save

and $MPC = \frac{dc}{dy}$ and $MPS = \frac{ds}{dy}$

but, $Y = C + S$

$$\Rightarrow \frac{dy}{dy} = \frac{dc}{dy} + \frac{ds}{dy}$$

$$\frac{dc}{dy} + \frac{ds}{dy} = 1 \Rightarrow \boxed{MPC + MPS = 1}$$ (مهم)

Example:) If the Consumption function is $C = 0.01Y^2 + 0.2Y + 50$ then

① Find MPC when $Y = 30$

$$MPC = \frac{dc}{dy} = \frac{2}{100}Y + \frac{2}{10}$$

$$MPC\Big|_{Y=30} \rightarrow \frac{2}{100}(30) + \frac{2}{10} = \frac{60}{100} + \frac{2}{10}$$

$$= \frac{60}{100} + \frac{20}{100} = 0.80$$

② find MPS when $Y = 30$

$$MPS + MPC = 1$$

$$MPS + 0.80 = 1$$

$$MPS = 0.20$$

③ Find the formula for S in terms of Y

$$Y = S + C$$

$$S = C - Y$$

$$S = 0.01Y^2 + 0.2Y + 50 - Y$$

$$S = 0.01Y^2 - 0.8Y + 50$$

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ex) If the consumption function is $C = 0.01Y^2 + 0.2Y + 50$ calculate: MPc and MPs when $y = 30$

* بما انه معطيني اقتران الاستهلاك فبيدأ ب MPc

حل

$$MPc = \frac{dc}{dY} = \frac{2}{100}Y + \frac{2}{10}$$

$$MPc\Big|_{Y=30} \longrightarrow \frac{2}{100}(30) + \frac{2}{10} = 0.80$$

$$\Rightarrow MPs + MPc = 1$$

$$MPs + 0.80 = 1$$

$$MPs = 0.20$$

* $(\log_b f(x))' = \frac{1}{\ln b} * \frac{f'(x)}{f(x)}$

مشتقة اللوغاريتم

* $(\ln f(x))' = \frac{f'(x)}{f(x)}$

مشتقة ال ln

* $(e^{f(x)})' = f'(x) * e^{f(x)}$

* $(b^{f(x)})' = \ln b * f'(x) * b^{f(x)}$

b: رقم ثابت

ex) $f(x) = \log_5 2x^2$

حل

$$f'(x) = \frac{1}{\ln 5} * \frac{4x}{2x^2}$$

$$= \frac{1}{\ln 5} * \frac{2}{x}$$

ex) find the $f'(x)$.

① $f(x) = \ln 4x-1$

ك $f'(x) = \frac{4}{4x-1}$

② $f(x) = 2e^{x^3}$

ك $f'(x) = 3x^2 (2e^{x^3})$

$= 6x^2 e^{x^3}$

③ $f(x) = 5^{2x}$

ك $f'(x) = \ln 5 * (2) * (5^{2x})$

$= 2\ln 5 * (5^{2x})$

④ $y = e^x \cdot \ln x$

$y' = (e^x \cdot \frac{1}{x}) + \ln x \cdot e^x (1)$

$= (\frac{e^x}{x} + \ln x \; e^x)$ * نأخذ e^x كعامل مشترك

$= e^x (\frac{1}{x} + \ln x)$

⑤ $y = \ln x$

$y' = \frac{1}{x}$

⑥ $y = e^x$

$y' = e^x$

ex) find the y' and $y'(1)$

① $y = e^{(x^2-7x)}$

$y' = (2x-7)\, e^{(x^2-7x)}$

$y'(1) = (2(1)-7)\, e^{(1^2-7)}$

$= -5\, e^{-6}$

$= \frac{-5}{e^6}$

لا تغير أهدافك وفقاً
لظروفك بل غير ظروفك
وفقاً لأهدافك

ex) find the y'

① $y = \ln(x^4 - 3x)$

حل $y' = \frac{4x^3 - 3}{x^4 - 3x}$

② $y = \ln\left(\frac{x^3+5x}{x^2-3x}\right)$

الحل $y = \ln(x^3+5x) - \ln(x^2-3x)$

$y' = \frac{3x^2+5}{x^3+5x} - \frac{2x-3}{x^2-3x}$

③ $y = e^x (x^3 - 2x)^7$

مشتقة الضرب

$y' = e^x(7(x^3-2x)^6(3x^2-2) + e^x(x^3-2x)^7$

④ If $h = (\ln x)(x^2-3)^8$ find h'

$h' = (\ln x)(8(x^2-3)^7(2x)) + (x^2-3)^8 * \frac{1}{x}$

ex) $y = \ln\left(\sqrt{\frac{5x+4}{x^2-6}}\right)$. find y'

sol

$y = \ln\left(\frac{5x+4}{x^2-6}\right)^{\frac{1}{2}}$

$= \frac{1}{2}\ln\left(\frac{5x+4}{x^2-6}\right)$

$= \frac{1}{2}\left(\ln(5x+4) - \ln(x^2-6)\right)$

$y' = \frac{1}{2}\left(\frac{5}{5x+4} - \frac{2x}{x^2-6}\right)$

ex) find the y'

① $y = \ln 9$

$y' = 0$

② $f(x) = (x^2+2x)\ln(x^2+2x)$

sol

$f'(x) = (x^2+2x)\left(\frac{2x+2}{x^2+2x}\right) + \ln(x^2+2x)(2x+2)$

③ $y = \ln(x^2+2x+1)$

$y' = \frac{2x+2}{x^2+2x+1}$

④ $y = \frac{\ln x}{x+2}$

$y' = \frac{(x+2)\left(\frac{1}{x}\right) - \ln x\,(1)}{(x+2)^2}$

$= \frac{\frac{x+2}{x} - \ln x}{x^2+4x+4}$

* قوانين وقواعد الـ ln

1. $\ln(xy) = \ln x + \ln y$
2. $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$
3. $\ln x^{n} = n \ln x$
4. $\ln e = \frac{\log e}{\log e} = 1$

كدم الإكتراث
بالمنافقين
هو أقسى انتقام منهم

** قواعد (قوانين) الـ (e)

1. $e^{x} \cdot e^{y} = e^{x+y}$
2. $\frac{e^{x}}{e^{y}} = e^{x-y}$
3. $e^{\ln x} = x$
4. $\ln e^{x} = x \underbrace{\ln e}_{(1)} = x$

*** قواعد (قوانين) الـ (log)

1. $\log_a (xy) = \log_a x + \log_a y$
2. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_a x^{n} = n \log_a x$
4. $2^{5} = 32 \Rightarrow \log_2 (32) = 5$
5. $\log x = \log_{10} x$
6. $\log_a a = 1$
7. $\log_5 1 = 0 \Rightarrow 5^{0} = 1$

ex) $y = \ln (x^2+3)^4 = 4 \ln (x^2+3)$

$y' = \frac{(4)(2x)}{x^2+3} = \frac{8x}{x^2+3}$

ex) find y'

$y = \ln (x^2 \cdot \sqrt{x^3+3x})^x$

$= x \ln (x^2 \cdot \sqrt{x^3+3x})$

$y = x \left(\ln x^2 + \ln (x^3+3x)^{\frac{1}{2}} \right)$

$y = x \left(2 \ln x + \frac{1}{2} \ln (x^3+3x) \right)$

$= x \left(2 \left(\frac{1}{x}\right) + \frac{1}{2} \frac{3x^2+3}{x^3+3x} \right) + (1) \left(2 \ln x + \frac{1}{2} \ln (x^3+3x) \right)$

ex) find y'

$y = \ln (x(x+1))^4 = 4 \ln (x(x+1))$

$y = 4 (\ln x + \ln x+1)$

$y' = 4 \left(\frac{1}{x} + \frac{1}{x+1} \right)$

ex) find y'

$y = \ln \frac{x}{\sqrt{x+3}}$

$y = \ln x - \ln \sqrt{x+3}$

$y = \ln x - \ln (x+3)^{\frac{1}{2}}$

$= \ln x - \frac{1}{2} \ln (x+3)$

$y' = \frac{1}{x} - \frac{1}{2} \frac{1}{x+3}$

ex) find if $f(x) = \ln\left(\frac{\sqrt{x+3}}{x^2+4}\right)^3$

$f(x) = 3\left(\ln \frac{\sqrt{x+3}}{x^2+4}\right)$

لا تكتب ، فاهمتها بعد اصفرار الزرع يأتيك الربيع

$= 3(\ln\sqrt{x+3} - \ln x^2+4)$

$= 3(\ln(x+3)^{\frac{1}{2}} - \ln(x^2+4))$

$= 3(\frac{1}{2}\ln(x+3) - \ln(x^2+4))$

$f'(x) = 3(\frac{1}{2}\frac{1}{x+3} - \frac{2x}{x^2+4})$

ex) If $f(x) = x^2 \ln x$ then $f'(x)$

الحل

$f'(x) = x^2 \frac{1}{x} + (2x)\ln x$

$= x + 2x\ln x$

$= x(1+2\ln x)$ x عامل مشترك

ex) $y = \frac{1}{x^4}$ and $y' = -16$ then $x = ??$

$y = \frac{1}{x^4} = x^{-4}$

$y' = \frac{-4x^{-5}}{-4} = \frac{-16}{-4}$

$x^{-5} = 4$

$\frac{1}{x^5} = \frac{4}{1}$

$4x^5 = 1$

$x^5 = \frac{1}{4}$

$x = \sqrt[5]{\frac{1}{4}}$

ex) find the y'

① $y = \ln(x(x+1)^4)$

كل هذا التعب سيزول
في سبيل تحقيق ما أريد

حل

$y = \ln x + \ln(x+1)^4$

$y = \ln x + 4\ln(x+1)$

$y' = \frac{1}{x} + \frac{4}{x+1}$

* equation of the tangent line   معادلة خط المماس

$y - y_0 = f'(x)|_{x_0} (x - x_0)$

ex) find the tangent line of $x=3$ for $f(x) = x^2$
($x = 3$ ↓ x_0)

حل

$y - y_0 = f'(x)|_{x_0} (x - x_0)$

$f(x) = y_0$

$y - 9 = 6(x - 3)$

$f(3) = 3^2 = 9$

$y - 9 = 6x - 18$

$f'(x) = 2x$
$f'(3) = 6$

$y = 6x - 9$
↓
y موضوع القانون

* The Chain Rule   قاعدة السلسلة

$$y = (f(x))^n \text{ then } \frac{dy}{dx} = n(f(x))^{n-1} * f'(x)$$

ex) $y = (3x^2 - 5x + 2)^4$

حل $y' = 4(3x^2 - 5x + 2)^3 * (6x - 5)$

ex) find the y'

① $y = \sqrt{1 + x^2}$

$y = (1 + x^2)^{\frac{1}{2}}$

$y' = \frac{1}{2}(1 + x^2)^{-\frac{1}{2}} * 2x$

* الإشتقاق الضمني   ومن متطلبات CH4

$$xy^2 + y^3 = 2x, \text{ find } \boxed{\frac{dy}{dx}}$$

يكلخ طريقتان

① الإشتقاق الضمني

② الإشتقاق الجزئي CH5

الإشتقاق الضمني: عند اشتقاق y نضربه بـ $\frac{dy}{dx}$

مثال ex) $xy^2 + y^3 = 2x$, find $\frac{dy}{dx}$

سوف نحله هنا على الإشتقاق الضمني

الحل

$$x(2y)\left(\frac{dy}{dx}\right) + y^2(1) + 3y^2\left(\frac{dy}{dx}\right) = 2$$

$$2xy\left(\frac{dy}{dx}\right) + y^2 + 3y^2\left(\frac{dy}{dx}\right) = 2$$

* بنخلي الحدود إلي فيها $\frac{dy}{dx}$ مع بعض

$$2xy\left(\frac{dy}{dx}\right) + 3y^2\left(\frac{dy}{dx}\right) = 2 - y^2$$

* نسحب عامل مشترك $\frac{dy}{dx}$

$$\frac{dy}{dx}\left(2xy + 3y^2\right) = 2 - y^2$$

* نجعل $\frac{dy}{dx}$ موضوع القانون

$$\frac{dy}{dx} = \frac{2 - y^2}{2xy + 3y^2}$$

تقديم الوظيفة بتصرفاتك
أفضل من تقديمها بشفتيك

بيكسار

واستمر فيها

stationary Point نقطة حرجة
critical Point
optimization of economic function } → Max, Min for function (عبارة عن) (Max: قيمة عظمى, Min: قيمة صغرى)

- optimization of economic function :-

To find Maximum or Minimum value of $y = f(x)$:-

① we find stationary Points of the function of by writing $f'(x) = 0$

② we find $f''(x)$ and then, at a st. Point $x = a$

* (a) if $f''(a) > 0$. then at a, we have a Minimum value for $f(x)$
* (a) if $f''(a) < 0$. then at a, we have a Maximum value for $f(x)$

Minimize, Minimum, Maximize, Maximum :- إذا كان السؤال طالب - stationary

Q) $f(x) = 2x^2 - x$, find the stationary Point. (classify)

ح/ ① $f'(x) = 4x - 1$

② $f'(x) = 0$

$4x - 1 = 0$

$x = \frac{1}{4}$ → ماهي نقطة stationary X هي

هل النقطة دي أعظم ولا أصغر — Max or min

③ $f''(x) = 4$

④ عوض في الخطوة الثانية في خطوة ثلاثة → $f''(\frac{1}{4}) = 4 > 0$

function at $x = \frac{1}{4}$ has a min

- إذا طلب stationary Point هات X اللي طلع بعوض X في y أو الاقتران

$f(x) = 2x^2 - x = 2(\frac{1}{4})^2 - \frac{1}{4} = 2(\frac{1}{16}) - \frac{1}{4} = \frac{1}{8} - \frac{1}{4} = \frac{-1}{8}$

$(\frac{1}{4}, \frac{-1}{8})$

* اذا كانت ناتج تعويض نقطة X بالمشتقة الثانية $> 0 \Leftarrow$ Minimum الاقتران بحالة

* = = : : : : $< 0 \Leftarrow$ الاقتران بحالة Maximum

* = : : : : : $= 0$ Can't classify

Q.) find stationary Point for $f(x) = 2x^3 + 3x^2 - 12x + 4$ and then find Maximum and Minimum value of $f(x)$.

حل

① $f'(x) = 6x^2 + 6x - 12$

② $f'(x) = 0$

Point (x, y)
Value (y)

$\frac{1}{6} * (6x^2 + 6x - 12 = 0)$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2 \ / \ x = 1$

→ stationary Point are $[-2, 1]$

Now:-

$f''(x) = 12x + 6$

* At $x = -2$

$f''(-2) = 12(-2) + 6 = -18 < 0$

⟹ At $x = -2$, we have a maximum value for $f(x)$

Maximum value is $f(-2)$

$2(-2)^3 + 3(-2)^2 - 12(-2) + 4$

$2(-8) + 3(4) + 24 + 4 = 24$

⟹ At $x = 1$

$f''(1) = 12(1) + 6 = 18 > 0$

we have minimum value $f(1) = 2(1)^3 + 3(1)^2 - 12(1) + 4$

$= -3$

ex) Find and classify the stationary Point(s) ?

① $f(x) = 3x^2 + 12x - 35$

$f'(x) = 6x - 12$

$6x - 12 = 0$

$6x = 12$

$x = 2$

$f''(x) = 6$

$f''(2) = 6 > 0$

function at $x = 2$ has a minimum.

$f(2) = 3(2)^2 + 12(2) - 35 = 1$

$(2, 1)$

② $f(x) = -2x^3 + 15x^2 - 36x + 27$

$f'(x) = -6x^2 + 30x - 36$

$\frac{1}{-6} \times (-6x^2 + 30x - 36 = 0)$

حاول تكمل بقية الحل بنفسك

$x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

$x = 3 \;/\; x = 2$

$f''(x) = -12x + 30$

At $x = 3$

$f''(3) = -12(3) + 30 = -36 + 30 = -6 < 0$

At $x = 3$, we have a maximum value for $f(x)$

Maximum value is $f(3)$

$f(3) = -2(3)^3 + 15(3)^2 - 36(3) + 27$

$= -54 + 135 - 108 + 27 = 0$

At $x = 2$, $f''(2) = -12(2) + 30 = 6 > 0$

At $x = 3$, we have a minimum value for $f(x)$

Minimum value is $f(2)$

$f(2) = -2(2)^3 + 15(2)^2 - 36(2) + 27$

$= -2(8) + 15(4) - 72 + 27 = -1$

Ex) Find and classify the stationary Point(s)?

① $f(x) = x e^{-x}$

حل $f'(x) = x(-e^{-x}) + (1)e^{-x}$

$= -x e^{-x} + e^{-x}$

$-x e^{-x} + e^{-x} = 0$

$\frac{-x e^{-x}}{-e^{-x}} = \frac{-e^{-x}}{-e^{-x}}$

$x = 1$

$f''(x) = (-x) - e^{-x} + e^{-x}(-1) - e^{-x}$

$= x e^{-x} + e^{-x} e^{-x}$

$= x e^{-x} + e^{-2x}$

$f''(1) = 1 e^{-1} + e^{-2}$

$= \frac{1}{e} + \frac{1}{e^2}$ > 0

function at $x = 1$ has a minimum

$f(1) = 1 e^{-1} = \frac{1}{e}$

② $f(x) = \ln x - x$

حل $f'(x) = \frac{1}{x} - 1$

$\frac{1}{x} - 1 = 0$

$\frac{1}{x} = 1$

$\boxed{x = 1}$

$f''(x) = -x^{-2} = \frac{-1}{x^2}$

$f''(1) = \frac{-1}{1} = -1 < 0$ function at $x = 1$ has a maximum

$f(1) = \ln 1 - 1 = -1$

(27)

EX) $P + Q = 30$ level of output the Maximum TR.

نجعل P موضوع القانون — معناها جد قيمة Q

$P = 30 - Q$

$TR = P \cdot Q$

$= (30 - Q)(Q)$

$= 30Q - Q^2$

$(TR)' = 30 - 2Q$

$(TR)' = 0$

$30 - 2Q = 0$

$\boxed{Q = 15}$

جابلنا قيمة وحدة
وهو محددلنا انها
Max ماخ داعي
أكمل الخطوات
التأكد من max او min

at $Q = 15$, TR Max

EX) Total cost function $TC = Q^2 + 3Q + 36$ calculate level of output (Q) that minimize Average cost AC

حل $AC = \frac{TC}{Q} = \frac{Q^2 + 3Q + 36}{Q} = Q + 3 + \frac{36}{Q}$

$AC = Q + 3 + 36Q^{-1}$

$(AC)' = 1 - \frac{36}{Q^2}$

$1 - \frac{36}{Q^2} = 0 \Rightarrow \boxed{Q = 6}$ is a st. Point

(Now), $(AC)'' = -36(-2)(Q^{-3}) = \frac{72}{Q^3}$

$(AC)''(6) = \frac{72}{216} > 0$

أكمل الخطوات *
ماخ داعي أكمل
لأن نقطة وحدة وهو طالب Min فأكيد النقطة هي Min
وهو طالب قيمة Q

$\Rightarrow$ The have minimum value of the AC at $Q = 6$.

$\Rightarrow$ Minimum value the AC at $Q = 6$ is $AC = Q + 3 + \frac{36}{Q}$

$= 6 + 3 + 6 = 15$

Ex) Production function $Q = 6L^2 - 0{,}2L^3$

① find the size of work force (L) that Maximize outPut.

ملاحظة

Product = Q = outPut = TP

Average Product $AP = \frac{TP}{L}$

$Q = 6L^2 - 0{,}2L^3$

مشتقة اولى بالاعتبار $Q' = 12L - 0{,}6L^2$

$12L - 0{,}6L^2 = 0$

$L(12 - 0{,}6L) = 0$

$L = 0 \;/\; 12 - 0{,}6L = 0$

$\frac{12}{0{,}6} = \frac{0{,}6L}{0{,}6}$

$L = 20$

المشتقة الثانية $Q'' = 12 - 1{,}2L$

$L = 0$

$12 - 1{,}2(0)$

$12 > 0 \rightarrow$ Minimum

نستثني الصفر قيمة ال (L)

Max بالطبع

$L = 20 \rightarrow 12 - 1{,}2(20) < 0 \rightarrow$ Maximum

Maximum when $L = 20$

② find the size of work force (L) that Maximum Average Product of Labor

الحل $AP = \frac{TP \rightarrow Q}{L} = \frac{6L^2 - 0{,}2L^3}{L} = 6L - 0{,}2L^2$

$(AP)' = 6 - 0.4L$

$6 - 0.4L = 0$

$6 = 0.4L \Rightarrow L = 15$

$(AP)'' = -0{,}4$

$(AP)''(15) = -0{,}4 < 0$ Maximum AP when $L = 15$

Ex) If the demand function = $P + Q = 30$ and $TC = \frac{1}{2}Q^2 + 6Q + 7$

① find level of output (Q) that Maximize the Profit π.

② The Maximum Profit

الحل ① $\pi = TR - TC$

$TR = P \cdot Q \quad = (30 - Q)(Q)$

$TR = 30Q - Q^2$

$\Rightarrow \pi = (30Q - Q^2) - (\frac{1}{2}Q^2 + 6Q + 7)$

$= 30Q - Q^2 - \frac{1}{2}Q^2 - 6Q - 7$

$\pi = \frac{-3}{2}Q^2 + 24Q - 7$

$(\pi)' = -3Q + 24$

$(\pi)' = 0$

$-3Q + 24 = 0$

$-3Q = -24$

$Q = 8$

دوّر قوليك ركه
ارضا بكلشي

Now $(\pi)''(Q) = -3 < 0$

$(\pi)''(8) = -3 < 0 \Rightarrow$ we have a maximum Profit

π at $Q = 8$

② Maximum Profit is

$\pi(8) = \frac{-3}{2}Q^2 + 24Q - 7$

$= \frac{-3}{2}(8)^2 + 24(8) - 7$

$= -96 + 192 - 7$

$96 - 7 = \boxed{89}$

أسئلة سنوات سابقة

ex

1) The fixed cost is 200 and variable costs is given by $VC = 2Q - 7$. The average cost when $Q = 4$ units equals =

① 55 ② 50 ③ 56.25 ④ 60 ⑤ 58

2) $f'(x) = x^2 - 4x + 3$ then $f(x)$ has a minimum at =

① $x = -3$ ② $x = -1$ ③ $x = 2$ ④ $x = 1$ ⑤ $x = 3$

3) $f(x) = \ln(x^2+1)$ then $f'(x)$ equals =

① $\frac{1}{x^2+1}$ ② $\frac{x^2}{x^2+1}$ ③ $\frac{x}{x^2+1}$ ④ $\frac{2x}{x^2+1}$ ⑤ $\frac{1}{\ln(x^2+1)}$

4) let $y = \sqrt[3]{x^2}$. then $\frac{dy}{dx}$ at $x = 8$ equals

① 3 ② $\frac{1}{3}$ ③ $\frac{-1}{3}$ ④ $\frac{-2}{3}$ ⑤ $\frac{2}{3}$

5) $f(x) = x\sqrt{x^2+x+1}$ then $f'(x)$ equals =

6) If the consumption function is $C = 50 + 3\sqrt{Y}$ then the margenal Propensity to save MPs at $Y = 100$

7) The slope tangent of $f(x) = x^2 + 1$ at the Point $(1, 2)$ is

8) $Y = e^{3x}$ then $\frac{d^2y}{dx^2}$ equals

(المشتقة الثانية)

9) At equilibrium the tax t Per unit given by $t = 80 - 10Q$. The total tax revenue is given by

① $8t - t^2$ ② $80t - \frac{t^2}{10}$ ③ $8 - \frac{t}{10}$

④ $80t - \frac{t}{10}$ ⑤ $8t - \frac{t^2}{10}$

Q) At equilibrium the tax t per unit given by $t = 80 - 10Q$. The total tax revenue is given by

* Total tax revenue $= T * Q$

$$T = 80 - 10Q$$

جعل Q موضوع القانون

$$\frac{T - 80}{-10} = \frac{-10Q}{-10}$$

$$Q = \frac{-(T - 80)}{10}$$

$$Q = \frac{-T + 80}{10}$$

$$Q = 8 - \frac{T}{10}$$

Total tax revenue $= T \cdot Q$

$$= \left(8 - \frac{T}{10}\right) T$$

$$= 8T - \frac{T^2}{10}$$

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10) If $f(1)=2$, $g(1)=3$, $f'(1)=3$ and $g'(1)=2$

If $h(x)=\frac{f(x)}{g(x)}$ then $h'(1)$ equals :-

Sol

$$h(x)=\frac{f(x)}{g(x)}$$

$$h'(x)=\frac{g(x)\,f'(x)-f(x)\,g'(x)}{g(x)^2}$$

$$h'(1)=\frac{g(1)\,f'(1)-f(1)\,g'(1)}{g(1)^2}=\frac{3(3)-2(2)}{9}=\frac{5}{9}$$

11) If $y=\ln\left(x^2\sqrt{x-1}\right)$, then find $y'(-2)$

12) If the Average Cost function is $AC=\frac{10}{Q}+3Q-2$
then find the marginal Cost MC at $Q=1$

13) The slope tangent line to the curve $f(x)=\frac{1}{x^2+2}$ at $x=1$ is

Sol

$$f(x)=(x^2+2)^{-1}$$

$$f'(x)=-(x^2+2)^{-2}\,2x$$

$$f'(x)=\frac{-2x}{(x^2+2)^2}\Rightarrow f'(1)=\frac{-2}{9}$$

14) If the Average revenue $AR=40-3\sqrt{Q}$
Then the Marginal revenue function MR

Sol $TR=AR(Q)$

$$TR=40Q-3Q^{\frac{3}{2}}$$

$$MR=40-\frac{9}{2}\sqrt{Q}$$

15) $f(x) = e^{x^2}(x^3 + \ln x)^{10}$. then $f'(x)$

16) let the total revenue function $TR = \frac{Q^3}{3} - 3Q^2 + 5Q + 20$

A) The stationary values for TR are at Q = ... and Q = ...

B) The Minimum value for attains at Q = ... and the Maximum value for TR attains at Q = ...

17) $AC = \frac{6}{Q} + 3Q$ find Marginal cost MC at Q = 5

18) $C = 0.03y^2 - y + 100$ find MPs at Y = 20

$MPc = C'$

$MPc = 0.06y - 1$

$MPc \Big|_{y=20} = 0.06(20) - 1$

$1.2 - 1 = 0.2$

$MPs + MPc = 1$

$MPs + 0.2 = 1$

$\boxed{MPs = 0.8}$

19) $y = e^{5x}(x^5 + 2)$ find y' .

20) find the slope $y = 10x - 1$ is

Sol slope $= y' = 10$

21) find $f'(x)$, $f(x) = x^2 + 3x$

22) $f(x) = \ln(x)^2 - x$ find $f''(3)$

23) let Total revenue $TR = \frac{-Q^3}{3} - 3Q^2 + 5Q + 2$

A) Stationary Value are $Q =$ ____ and $Q =$ ____

B) The Minimum Value at $Q =$ ____

The Maximum Value at $Q =$ ____

24) $\frac{2}{2-x} = y$ find y' ?

25) $f(x) = \ln\left(\frac{(2x+3)^5}{e^{3x}}\right)$ $F'(1)$

26) if the Production $Q = 100\sqrt{L} + 2L$ then MPL when $L = 25$

27) $f(x) = 2x - \sqrt{(2x^2+6)}$ then the slope of the tangent at $x = 1$ is .

28) let $\pi = \frac{Q^3}{3} + \frac{Q^2}{2} - 6Q + 8$ then π has

a) Max at $Q = -3$ b) Max at $Q = 2$

c) Min at $Q = -3$ D) Min at $Q = 2$ e) None

29) $f(x) = 3x + \sqrt[3]{(2x^2+6)}$ then the slope tangent at $x = 1$ is

30) $y = \frac{1}{x^2}$, If $y'(a) = 54$ find a

sol

$y' = \frac{-2}{x^3}$

$\frac{-2}{x^3} = 54 \Rightarrow x^3 = \frac{-2}{54}$

$x = \sqrt[3]{\frac{-2}{54}}$

30) The demand equation is $P = 120 - 2Q$, then the MR at $Q = 10$ is

31) $f(x) = (x^2 + x + 1)^3$, then $f'(1) =$

32) $\ln x = 6$, $\ln y = 10$ then $\ln \frac{\sqrt{x}}{y} =$

sol

$\ln \sqrt{x} - \ln y$

$\ln x^{\frac{1}{2}} - \ln y$

$\frac{1}{2} \ln x - \ln y$

$\frac{1}{2}(6) - 10 = 3 - 10 = -7$

33) $y = 1 + e^{2x}$ then $x =$

a) $\ln \sqrt{y-1}$ b) $\ln \sqrt{y-2}$ c) $\ln \sqrt[3]{y-1}$ D) $\ln \sqrt[3]{y-2}$

e) None

34) $e^{2\ln 2x} = e^{\ln (2x)^2} = (2x)^2 = 4x^2$

35) $f(x) = x(x-2)(x-3)$, then find $f''(3)$.

sol $f(x) = x^3 - 5x^2 + 6x$

$f'(x) = 3x^2 - 10x + 6$

$f''(x) = 6x - 10$

$f''(3) = 6(3) - 10 = 18 - 10 = 8$

36) $f(x) = 2x^3 + 3x^2 - 12x + 4$ find stationary value

sol $f'(x) = 6x^2 + 6x - 12$

$f'(x) = 0 \implies (6x^2 + 6x - 12 = 0) * \frac{1}{6}$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$\boxed{x = -2}$

$\boxed{x = 1}$

Chapter 5 (Partial Differentiation)

#بيكسلز
#لتكمل الصورة
#وأستعركم فيها

* function of two variable

* $Y = f(x)$ is a function, then it is a function of one variable
* $Z = f(x,y)$ is a function of two variables.

Ex) let $z = f(x,y) = xy^2 + 5xy^3 + 6x - 7$

① $f(2,1) = 2(1)^2 + 5(2)(1)^3 + 6(2) - 7$
$= 2 + 10 + 12 - 7 = 17$

② $f(0,-2) = 0(-2)^2 + 5(0)(-2)^3 + 6(0) - 7$
$= -7$

الآمال العظيمة
- صنع
الأشخاص العظماء

* Partial (جزئية) Derivatives (مشتقات) of $z = f(x,y)$ are :-

① $\frac{\partial z}{\partial x} = f_x$

② $\frac{\partial z}{\partial y} = f_y$

③ $\frac{\partial^2 z}{\partial x^2} = f_{xx}$

④ $\frac{\partial^2 z}{\partial y^2} = f_{yy}$

⑤ $f_{xy} = (f_x)_y$

⑥ $f_{yx} = (f_y)_x$

* فكرة الإشتقاق الجزئي
- أشتق المطلوب
- أي شيء غير مطلوب يعتبر ثابت

الثابت ← في حالة الجمع والطرح مشتقته عبارة عن صفر (zero)
← في حالة الضرب أو القسمة ينزل كما هو ثم نشتق الإقترانه

EX) Let $z = f(x,y) = x^3y^2 - xy + 2x + 3y - 10$

find : (all Partial derivatives) at the Point $(2,1)$

① $f_x = 3x^2y^2 - y + 2 + 0 - 0$

$= 3x^2y^2 - y + 2$

$\Rightarrow$ at $(2,1) = 3(2)^2(1)^2 - 1 + 2 = 13$

② $f_y = 2x^3y - x + 0 + 3 - 0$

$= 2x^3y - x + 3$

$\Rightarrow$ at $(2,1) = 2(2)^3(1) - (2) + 3 = 17$

③ $f_{xx} = (f_x)_x = (3x^2y^2 - y + 2)_x$

$= 6xy^2$

$\Rightarrow$ at $(2,1) = 6(2)(1)^2 = 12$

④ $f_{yy} = (f_y)_y = (2x^3y - x + 3)_y$

$= 2x^3$

$\Rightarrow$ at $(2,1) = 2(2)^3 = 16$

⑤ $f_{xy} = (f_x)_y = (3x^2y^2 - y + 2)_y$

$= 6x^2y - 1$

$\Rightarrow$ at $(2,1) = 6(2)^2(1) - 1 = 23$

⑥ $f_{yx} = (f_y)_x$

$= (2x^3y - x + 3)_x$

$= 6x^2y - 1$

$\Rightarrow$ at $(2,1) = 6(2)^2(1) - 1 = 23$

* $f(x,y) = x^3y^2 - xy + 10$

له بدل كام (under $f(x,y)$) — متغيرين (under x, y)

استخدام كام

③

ex) $f(x,y) = x^3y + \ln xy + e^{x^2y}$

find f_x and f_{xx}

حل ① $f_x = 3x^2y + \frac{\cancel{x}}{x\cancel{x}} + 2xy\,e^{x^2y}$

② $f_{xx} = 6xy + \frac{-1}{x^2} + 2xy \cdot 2xy\,e^{x^2y} + e^{x^2y}(2y)$

$= 6xy - \frac{1}{x^2} + 4x^2y^2\,e^{x^2y} + 2y\,e^{x^2y}$

ex) $f(x,y) = 3x^2y + y^2$

find $f_{yx}(1,1)$.

* من كلم نغير معادلة الحياة سبق بكانه كما هو

$f_y = 3x^2 + 2y$

$f_{yx} = 6x$

$f_{yx}(1,1) = 6(1) = 6$

ex) $f(x,y) = \frac{x}{y+2} + \ln xy$ then f_x

حل

$f_x = \frac{1}{y+2} + \frac{\cancel{x}}{y\cancel{x}}$

$= \frac{1}{y+2} + \frac{1}{x}$

ex) $f(x,y) = \frac{\sqrt{x}}{y} - \frac{y^{\frac{2}{3}}}{x^2} + 5xy$ find f_x and $f_x(4,3)$

حل $f(x,y) = \frac{x^{\frac{1}{2}}}{y} - y^{\frac{2}{3}}(x^{-2}) + 5xy$

$f_x = \frac{x^{-\frac{1}{2}}}{2y} - (-2x^{-3})\,y^{\frac{2}{3}} + 5y$

$= \frac{1}{2\sqrt{x}\cdot y} + \frac{2\sqrt[3]{y^2}}{x^3} + 5y$

$f_x(4,3) = \frac{1}{2\sqrt{4}\,(3)} + \frac{2\sqrt[3]{(3)^2}}{4^3} + 5(3)$

$\Longrightarrow f_x(4,3) = \frac{1}{12} + \frac{\sqrt[3]{9}}{32} + 15$

قاعدة → $f_{xy} = f_{yx}$

ex) $z = f(x,y) = e^{xy}$ find all Partial derivatives of f.

① $f_x = y e^{xy}$

② $f_y = x e^{xy}$

③ $f_{xx} = y(y e^{xy}) = y^2 e^{xy}$

④ $f_{yy} = x(x e^{xy}) = x^2 e^{xy}$

⑤ $f_{xy} = y(x e^{xy}) + e^{xy}(1) = xy e^{xy} + e^{xy}$

⑥ $f_{yx} = x(y e^{xy}) + e^{xy}(1) = xy e^{xy} + e^{xy}$

ex) find all Partial derivatives

① $f(x,y) = y e^{x}$

*) $f_x = y e^{x}$

*) $f_y = e^{x}$

*) $f_{xx} = y e^{x}$

*) $f_{yy} = 0$

*) $f_{xy} = e^{x}$

*) $f_{yx} = e^{x}$

$\sqrt[\text{خارج}]{x^{\text{داخل}}} = x^{\frac{\text{داخل}}{\text{خارج}}} = x^{\frac{1}{2}}$

2) $f(x,y) = \sqrt[3]{x}\ \sqrt[4]{y^3}$

حل $f(x,y) = x^{\frac{1}{3}}\ y^{\frac{3}{4}}$

*) $f_x = \frac{1}{3} x^{\frac{-2}{3}}\ y^{\frac{3}{4}} = \frac{\sqrt[4]{y^3}}{3\sqrt[3]{x^2}}$

*) $f_y = \frac{3}{4} y^{\frac{-1}{4}}\ x^{\frac{1}{3}} = \frac{3\sqrt[3]{x}}{4\sqrt[4]{y}}$

*) $f_{xx} = \frac{1}{3}\left(\frac{-2}{3}\right) x^{\frac{-5}{3}}\ y^{\frac{3}{4}} = \frac{-2}{9} x^{\frac{-5}{3}}\ y^{\frac{3}{5}} = \frac{-2\sqrt[5]{y^3}}{9\sqrt[3]{x^5}}$

*) $f_{yy} = \frac{3}{4}\left(\frac{-1}{4}\right) y^{\frac{-5}{4}}\ x^{\frac{1}{3}} = \frac{-3}{16} y^{\frac{-5}{4}}\ x^{\frac{1}{3}} = \frac{-3\sqrt[3]{x}}{16\sqrt[4]{y^5}}$

*) $f_{xy} = \frac{1}{3} x^{\frac{-2}{3}}\left(\frac{3}{4} y^{\frac{-1}{4}}\right) = \frac{1}{4}\ x^{\frac{-2}{3}}\ y^{\frac{-1}{4}} = \frac{1}{4\sqrt[3]{x^2}\sqrt[4]{y}}$ (with $\frac{3}{12}$ crossed out)

*) $f_{yx} = \frac{3}{4} y^{\frac{-1}{4}}\left(\frac{1}{3} x^{\frac{-2}{3}}\right) = \frac{1}{4}\ x^{\frac{-2}{3}}\ y^{\frac{-1}{4}} = \frac{1}{4\sqrt[3]{x^2}\sqrt[4]{y}}$ (with $\frac{3}{12}$ crossed out)

(5)

Q) find $\frac{dy}{dx}$, $xy^2 + y^3 = 2x$

← اشتقاق CH4

متغيرين الأوّل أمل كل CH5

* يُحل في CH4 كالاشتقاق الضمني

* الحل مثل هيك سؤال في CH5 بحله كل هالقاعدة

$$\frac{dy}{dx} = \frac{-f_x}{f_y}$$ → هناك شرط انه :- ناتج الاقتران = ثابت

الحل

أول خطوة $xy^2 + y^3 - 2x = 0$ اجعل ناتج الاقتران = ثابت

ثاني خطوة $f_x = y^2 - 2$

$f_y = 2xy + 3y^2$

ثالث خطوة $\frac{-(y^2-2)}{2xy+3y^2} = \frac{-y^2+2}{2xy+3y^2}$

ex) $e^{2x} + 6xy - 5x = 10$ find $\frac{dy}{dx}$

الحل

$$\frac{dy}{dx} = \frac{-f_x}{f_y}$$

$f_x = 2e^{2x} + 6y - 5$

$f_y = 6x$

$$\frac{dy}{dx} = \frac{-f_x}{f_y} = \frac{-(2e^{2x}+6y-5)}{6x} = \frac{-2e^{2x}-6y+5}{6x}$$

لا تهرب من مخاوفك
بل واجهها بقوة وتخلص منها

ex) find $\frac{dy}{dx}$

1) $x^2+\sqrt{xy}-3y=3$

الحل

$$f_x=2x+\frac{y}{2\sqrt{xy}}$$

$$f_y=\frac{x}{2\sqrt{xy}}-3$$

$$\frac{dy}{dx}=\frac{-f_x}{f_y}=\frac{-\left(2x+\frac{y}{2\sqrt{xy}}\right)}{\frac{x}{2\sqrt{xy}}-3}=\frac{-2x-\frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}}-3}$$

2) $xy+y^{\frac{-2}{3}}=y^2$

الحل

أول خطوة

ناتج اقتران = ثابت

$$xy+y^{\frac{-2}{3}}-y^2=0$$

$$f_x=y$$

$$f_y=x+\left(\frac{-2}{3}y^{\frac{-5}{3}}\right)-2y=x-\frac{2}{3\sqrt[3]{y^5}}-2y$$

$$\frac{dy}{dx}=\frac{-f_x}{f_y}=\frac{-y}{x-\frac{2}{3\sqrt[3]{y^5}}-2y}$$

3) $e^{xy}=2-\ln(xy)$

الحل

أول خطوة : ناتج الاقتران = ثابت

$$e^{xy}+\ln(xy)=2$$

$$f_x=ye^{xy}+\frac{y}{xy}=ye^{xy}+\frac{1}{x}$$

$$f_y=xe^{xy}+\frac{x}{xy}=xe^{xy}+\frac{1}{y}$$

$$\frac{dy}{dx}=\frac{-f_x}{f_y}=\frac{-\left(ye^{xy}+\frac{1}{x}\right)}{xe^{xy}+\frac{1}{y}}=\frac{-ye^{xy}-\frac{1}{x}}{xe^{xy}+\frac{1}{x}}$$

Unconstrained optimization

Let $z = f(x,y)$ be a function of two variables.

Then

نقاط حرجة

① we find $f_x = 0$ and $f_y = 0$ and so we find stationary points.

② If (a,b) is a stationary point then

(i) If $f_{xx}(a,b) > 0$, $f_{yy}(a,b) > 0$ and $f_{xx} \cdot f_{yy} - (f_{xy})^2 > 0$
then at (a,b) we have minimum at (a,b)

(ii) If $f_{xx}(a,b) < 0$ and $f_{yy}(a,b) < 0$ and $f_{xx} \cdot f_{yy} - (f_{xy})^2 > 0$
then we have Maximum at (a,b)

(iii) If $f_{xx}(a,b) \cdot f_{yy}(a,b) - (f_{xy}(a,b))^2 < 0$
then we have a saddle point at (a,b)

هُناك أشخاص
هُم الأمل بحد ذاته

لتستقبل الحياة [illegible]

بيكسلز

وأشعركم فيها

كافي في نفسك
لأنها تستحق

ex) $f(x,y) = 1000x + 800y - 2x^2 - 2xy - y^2$, find and classify the st. Point ?

الحل

① أول خطوة $f_x = 1000 - 4x - 2y = 0$

$f_y = 800 - 2x - 2y = 0$

بعدما تشتق ساويها بالصفر

② ثاني خطوة إيجاد قيمة x و y

$1000 - 4x - 2y = 0$
$800 - 2x - 2y = 0$
→ نستخدم طريقة الحذف أو التعويض

$1000 - 4x - 2y = 0 \Rightarrow 1000 - 4x - 2y = 0$

$-1 * (800 - 2x - 2y = 0) \Rightarrow -800 + 2x + 2y = 0$

$200 - 2x = 0$

$200 = 2x$

$\boxed{x = 100}$ عوضها بأحد المعادلات لإيجاد قيمة y

$800 - 2(100) - 2y = 0$

$800 - 200 - 2y = 0$

$600 - 2y = 0$

$600 = 2y$

$\boxed{y = 300}$

$(100, 300) \rightarrow$ St. Point

③ ثالث خطوة $f_{xx} = -4$

$f_{yy} = -2$

$f_{xy} = -2$

④ رابع خطوة $f_{xx}(100,300) = -4 < 0$ Ⓐ

$f_{yy}(100,300) = -2 < 0$ Ⓑ

$f_{xy}(100,300) = -2 < 0$

$f_{xx} \cdot f_{yy} - (f_{xy})^2 = 4 > 0$ Ⓒ

$\Rightarrow$ at $(100, 300)$ we have a maximum Point

مثال و حل

ex) Find and classify the stationary Point(s)?

① $f(x,y) = x^3 + y^3 - 3x - 3y$

الحل

خطوة اولى ① $f_x = 3x^2 - 3 = 0$

$f_y = 3y^2 - 3 = 0$

لا تخبر الناس عن خططك
أرهم نتائجك

خطوة ثانية ② إيجاد قيمة X و Y

$3x^2 - 3 = 0$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1 \Rightarrow x = 1 \qquad x = -1$

$3y^2 - 3 = 0$

$3y^2 = 3$

$y = \pm 1 \Rightarrow y = 1 \qquad y = -1$

st.Points $(1,1)$, $(1,-1)$, $(-1,1)$, $(-1,-1)$

ثالثة خطوة ③ $f_{xx} = 6x$

$f_{yy} = 6y$

$f_{xy} = 0$

رابع خطوة ④ عوض النقطة بناتج الاشتقاقات

At $(1,1)$

$f_{xx}(1,1) = 6 > 0$ Ⓐ

$f_{yy}(1,1) = 6 > 0$ Ⓑ

$f_{xy}(1,1) = 0$

$f_{xx} \cdot f_{yy} - (f_{xy})^2 = 6 \cdot 6 - 0 = 36 > 0$ Ⓒ

At $(1,1)$ we have a minimum Point

تابع حل السؤال

At (1, -1)

$f_{xx} = 6x$

$f_{yy} = 6y$

$f_{xy} = 0$

$f_{xx} = 6 > 0$ (A)

$f_{yy} = -6 < 0$ (B)

$f_{xx} \cdot f_{yy} - (f_{xy})^2$

$6 \cdot (-6) - 0 = -36 < 0$ (C)

At (1, -1) we have a saddle Point

At (-1, 1)

$f_{xx} = -6 < 0$ (A)

$f_{yy} = 6 > 0$ (B)

$f_{xx} \cdot f_{yy} - (f_{xy})^2$

$-6 \cdot (6) - 0 = -36 < 0$ (C)

At (-1, 1) we have a saddle Point

At (-1, -1)

$f_{xx} = -6 < 0$ (A)

$f_{yy} = -6 < 0$ (B)

$f_{xx} \cdot f_{yy} - (f_{xy})^2$

$(-6) \cdot (-6) - 0 = 36 > 0$ (C)

At (-1, -1) we have a maximum Point.

ex) find and classify the stationary Point(s) :-

* $f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 10$

$\therefore f_x = 3x^2 + 3y^2 - 6x = 0$

$f_y = 6xy - 6y = 0$ } أول خطوة

ثاني خطوة:- إيجاد قيمة x و y

$3x^2 + 3y^2 - 6x = 0$

$6xy - 6y = 0$

$6y(x-1) = 0$

$\Downarrow y = 0 \quad \Downarrow x = 1$

السر الحقيقي للنجاح
هو الحماسة المستمرة

If $x = 1$

$3x^2 + 3y^2 - 6x = 0$

$3 + 3y^2 - 6 = 0$

$3y^2 - 3 = 0$

$3y^2 = 3$

$(y^2 = 1)$ التخلص من التربيع - جذر للطرفين -

$y = \pm 1$

St. Points $(1,1)$, $(1,-1)$

If $y = 0$

$3x^2 + 3y^2 - 6x = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$\Downarrow x = 0 \quad \Downarrow x = 2$

St. Points $(0,0)$, $(2,0)$

St. Points $(1,1)$, $(1,-1)$, $(0,0)$, $(2,0)$

تابع حل السؤال ⇓

③ الخطوة

$f_{xx} = 6x - 6$

$f_{yy} = 6x - 6$

$f_{xy} = 6y$

السعادة لا تحتاج معجزات

④ الخطوة

- At (1,1)

$f_{xx} = 6(1) - 6 = 0$ Ⓐ

$f_{yy} = 6(1) - 6 = 0$ Ⓑ

$f_{xy} = 6$

$f_{xx} \cdot f_{yy} - (f_{xy})^2$

$(0) \cdot (0) - 6 < 0$ Ⓒ

At (1,1) we have a saddle Point

- At (1,−1)

$f_{xx} = 6(1) - 6 = 0$ Ⓐ $\qquad f_{xy} = -6$

$f_{yy} = 6(1) - 6 = 0$ Ⓑ

$f_{xx} \cdot f_{yy} - (f_{xy})^2 = (0) \cdot (0) - (-6)^2 = -36 < 0$ Ⓒ

At (1,−1) we have a saddle Point

- At (0,0)

$f_{xx} = 6(0) - 6 = -6 < 0$ Ⓐ $\qquad f_{xy} = 0$

$f_{yy} = 6(0) - 6 = -6 < 0$ Ⓑ

$f_{xx} \cdot f_{yy} - (f_{xy})^2 = (-6) \cdot (-6) - 0 = 36 > 0$ Ⓒ

At (0,0) we have a maximum Point.

- At (2,0)

$f_{xx} = 6(2) - 6 = 6 > 0$ Ⓐ $\qquad f_{xy} = 0$

$f_{yy} = 6(2) - 6 = 6 > 0$ Ⓑ

$f_{xx} \cdot f_{yy} - (f_{xy})^2 = (6) \cdot (6) - 0 = 36 > 0$ Ⓒ

At (2,0) we have a minimum Point

13

ex) find and classify the stationary Point(s).

$Z = f(x,y) = x^3 - 3x + xy^2$

Sol

① أول خطوة $f_x = 3x^2 - 3 + y^2 = 0$

$f_y = 2xy = 0$

② ثاني خطوة إيجاد قيمة x و y

$3x^2 - 3x + y^2 = 0$

$\frac{2xy}{2} = \frac{0}{2}$

سانِد نفسك بنفسك

$xy = 0$

$x = 0$ or $y = 0$

If $x = 0 \Rightarrow -3 + y^2 = 0$

$y^2 = 3$

$y = \pm\sqrt{3}$

st. Points $(0,\sqrt{3}), (0,-\sqrt{3})$

If $y = 0 \Rightarrow 3x^2 - 3 = 0$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

st. Points $(1,0), (-1,0)$

st. Points are $(0,\sqrt{3}), (0,-\sqrt{3}), (1,0), (-1,0)$

③ ثالث خطوة

$f_{xx} = 6x$

$f_{yy} = 2x$

$f_{xy} = 2y$

تابع حل السؤال !!

(4)

- At $(0,-\sqrt{3})$

$f_{xx} = 0$ Ⓐ $f_{xy} = 2(-\sqrt{3}) = -2\sqrt{3}$

$f_{yy} = 0$ Ⓑ

$f_{xx} \cdot f_{yy} - (f_{xy})^2$

$0 \cdot 0 - (-2\sqrt{3})^2 = -(4*3) = -12 < 0$ Ⓒ

at $(0,-\sqrt{3})$ we have a saddle Point

- At $(0,\sqrt{3})$

$f_{xx} = 0$ Ⓐ

$f_{yy} = 0$ Ⓑ

$f_{xy} = 2\sqrt{3}$

$f_{xx} \cdot f_{yy} - (2\sqrt{3})^2 = (0)\cdot(0) - (4*3) = -12 < 0$ Ⓒ

At $(0,\sqrt{3})$ we have a saddle Point.

- At $(1,0)$

$f_{xx} = 6 > 0$ Ⓐ $f_{xy} = 0$

$f_{yy} = 2 > 0$ Ⓑ

$f_{xx} \cdot f_{yy} - (f_{xy})^2$

$(6)*(2) - 0 > 0$

$12 > 0$ Ⓒ

At $(1,0)$ we have a minimum Point

- At $(-1,0)$

$f_{xx} = -6 < 0$ Ⓐ $f_{xy} = 0$

$f_{yy} = -2 < 0$ Ⓑ

$f_{xx} \cdot f_{yy} - (f_{xy})^2$

$(-6)*(-2) - 0$

$12 > 0$ Ⓒ

at $(-1,0)$ we have a Maximum Point

$X = Q_1$, $y = Q_2$ → زيادة

Profit

ex) $\pi = 1000Q_1 + 800Q_2 - 2Q_1^2 - 2Q_1Q_2 - Q_2^2$

find Maximum π $Q_1 = 100$, $Q_2 = 300$

Q_1, Q_2 → متغيرات

* بما انه طالب Max π فبعوض القيم في الاقترانة

مثلاً

* اذا كان طالب بالسؤال
find the value of Q that Max or min
بطلعله قيم Q_1 و Q_2

* اذا كان طالب
Max or Min (TR, TC, π)
بعد ما اطلع النقطة اللي تحقق (Max or Min)
نقطة (Q_1, Q_2) بعوضها بالاقترانة الأصلي

لا تيأس دمها ضاقت بك الحياة

ex) $TC = 2Q_1^2 + 2Q_1Q_2 + Q_2^2$

$TR_1 = 1000Q_1$

$TR_2 = 800Q_2$

* find Maximum Profit and Q_1, Q_2 Stationary Points.

حل Profit $= \pi = TR - TC$

$\pi = (TR_1 + TR_2) - TC$

$\pi = (1000Q_1 + 800Q_2) - (2Q_1^2 + 2Q_1Q_2 + Q_2^2)$

$= 1000Q_1 + 800Q_2 - 2Q_1^2 - 2Q_1Q_2 - Q_2^2$

① $\frac{\partial \pi}{\partial Q_1} = 0 \Rightarrow 1000 - 4Q_1 - 2Q_2 = 0$

② $\frac{\partial \pi}{\partial Q_2} = 0 \Rightarrow 800 - 2Q_1 - 2Q_2 = 0$

$1000 - 4Q_1 - 2Q_2 = 0$

$-1 * (800 - 2Q_1 - 2Q_2 = 0)$

$1000 - 4Q_1 - \cancel{2Q_2} = 0$

$-800 + 2Q_1 + \cancel{2Q_2} = 0$

$200 - 2Q_1 = 0$

$+2Q_1 \quad 2Q_1$

$200 = 2Q_1$

نعوضها بأحد الاشتقاقين $\boxed{Q_1 = 100}$

$1000 - 4(100) - 2Q_2 = 0$

$\boxed{Q_2 = 300}$

St. Point $(Q_1, Q_2) = 100, 300$

Max $\pi = 1000(100) + 800(300) - 2(100)^2 - 2(100)(300) - (300)^2 =$

للتأكد نشتق مرة ثانية $\frac{\partial^2 \pi}{\partial Q_1^2} = -4 < 0$ Ⓐ

$\frac{\partial^2 \pi}{\partial Q_2^2} = -2 < 0$ Ⓑ

$\frac{\partial^2 \pi}{\partial Q_1 Q_2} = -2$

$(-4) * (-2) - (-2)^2$ Ⓒ

$8 - 4 = 4 > 0$ Max { at (100, 300) we have Maximum Point

ex) The demand functions The demand functions for two types of goods are:-

$P_1 = 50 - Q_1$, $P_2 = 95 - 3Q_2$

and the total cost $TC = Q_1^2 + 3Q_1Q_2 + Q_2^2$.

find Q_1 and Q_2 that Maximizes the Profit.

sol

$\pi = TR - TC$ $\quad\quad TR = P \cdot Q$

$TR_1 = (50 - Q_1)Q_1 \Rightarrow 50Q_1 - Q_1^2$

$TR_2 = (95 - 3Q_2)Q_2 \Rightarrow 95Q_2 - 3Q_2^2$

$TR = (TR_1 + TR_2)$

$= (50Q_1 - Q_1^2 + 95Q_2 - 3Q_2^2)$

$\pi = 50Q_1 - Q_1^2 + 95Q_2 - 3Q_2^2 - (Q_1^2 + 3Q_1Q_2 + Q_2^2)$

$= 50Q_1 - Q_1^2 + 95Q_2 - 3Q_2^2 - Q_1^2 - 3Q_1Q_2 - Q_2^2$

$= 50Q_1 + 95Q_2 - 2Q_1^2 - 4Q_2^2 - 3Q_1Q_2$

$\frac{\partial \pi}{\partial Q_1} = 0 \Rightarrow 50 - 4Q_1 - 3Q_2 = 0$

$\frac{\partial \pi}{\partial Q_2} = 0 \Rightarrow 95 - 8Q_2 - 3Q_1 = 0$

$3\ (50 - 4Q_1 - 3Q_2 = 0)$

$-4\ (95 - 3Q_1 - 8Q_2 = 0)$

$150 - 12Q_1 - 9Q_2 = 0$

$-380 + 12Q_1 + 32Q_2 = 0$

$-230 + 23Q_2 = 0$

$23Q_2 = 230$

$\boxed{Q_2 = 10}$ ← [illegible]

st. Point (Q_1, Q_2)

$(5, 10)$

$50 - 4Q_1 - 3(10) = 0$

$20 - 4Q_1 = 0 \Rightarrow \frac{20}{4} = \frac{4Q_1}{4}$ $\quad \boxed{Q_1 = 5}$

ex) A Firm Produces the same Product of two factories. the Cost function for each factory are $TC_1 = 8Q_1$ and $TC_2 = Q_2^2$

The demand function for the good is $P = 100 - 2Q$ where $Q = Q_1 + Q_2$

find Q_1 and Q_2 which Maximize the Profit.

الحل $\pi = TR - TC$

$TR = P.Q$
$= (100 - 2Q)Q$
$= 100Q - 2Q^2$
$= 100(Q_1 + Q_2) - 2(Q_1 + Q_2)^2$

$TC = TC_1 + TC_2$
$= 8Q_1 + Q_2^2$

$TR = 100Q_1 + 100Q_2 - 2(Q_1^2 + 2Q_1Q_2 + Q_2^2)$

$\pi = 100Q_1 + 100Q_2 - 2Q_1^2 - 4Q_1Q_2 - 2Q_2^2 - (8Q_1 + Q_2^2)$
$= 100Q_1 + 100Q_2 - 2Q_1^2 - 4Q_1Q_2 - 2Q_2^2 - 8Q_1 - Q_2^2$
$= 92Q_1 + 100Q_2 - 2Q_1^2 - 4Q_1Q_2 - 3Q_2^2$

$\pi_{Q_1} = 92 - 4Q_1 - 4Q_2 = 0$
$\pi_{Q_2} = 100 - 4Q_1 - 6Q_2 = 0$

$-1(92 - 4Q_1 - 4Q_2 = 0)$
$100 - 4Q_1 - 6Q_2 = 0$

$-92 + 4Q_1 + 4Q_2 = 0$
$100 - 4Q_1 - 6Q_2 = 0$

$8 - 2Q_2 = 0$
$8 = 2Q_2$

كده بأجد الإحداثيات $\boxed{4 = Q_2}$

$92 - 4Q_1 - 4(4) =$
$92 - 4Q_1 - 16 = -4Q_1 + 76 = 0$
$76 = 4Q_1$
$\boxed{Q_1 = 19}$

S.t. Point is $(Q_1, Q_2) = (19, 4)$

(19)

* Constrained optimization

(objective / subject) Max or Min

ex) find the Minimum value of the object function

$Z = X^2 - 3XY + 12X$

Subject to the constraint $2X + 3Y = 6$.

الحل

① subject ← نجعل Y موضوع القانون

$2X + 3Y = 6$

$3Y = 6 - 2X$

$Y = 2 - \frac{2}{3}X$

② نعوض Y بال object

$Z = X^2 - 3X(2 - \frac{2}{3}X) + 12X$ → $Z = 3X^2 + 6X$

$Z = X^2 - 6X + 2X^2 + 12X$ → تحتوي على متغير واحد

لا نحتاجها لل CH4 (متغير واحد)

① $Z' = 6X + 6$

② $Z' = 0$

$6X + 6 = 0$

$6X = -6$

$X = -1$ → رأس الموضوع

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③ $Z'' = 6$

$Z''|_{X=-1} = 6 > 0$

at $X = -1$ function has minimum

$Y = 2 - \frac{2}{3}X$

$Y = 2 - \frac{2}{3}(-1)$

$Y = \frac{2}{1} + \frac{2}{3} = \frac{8}{3}$

نعوض X و Y بإقتران ال object

$Z = (-1)^2 - 3(-1)(\frac{8}{3}) + 12(-1)$

$Z = -3$ ⟹ Min value function object

ex) find the minimum value of a function $Z = -2x^2 + y^2$ objected of the constraint $y = 2x - 1$

Sol

$$Z = -2x^2 + (2x-1)^2$$

$$Z = -2x^2 + (4x^2 - 4x + 1)$$

$$Z = -2x^2 + 4x^2 - 4x + 1$$

$$Z = 2x^2 - 4x + 1$$

مشتقة اولى

$$\frac{dZ}{dx} = 4x - 4$$

$$\frac{dZ}{dx} = 0$$

$$4x - 4 = 0$$

$\boxed{x = 1}$ is a st. Point

مشتقة ثانية

Now, $\frac{d^2 Z}{dx^2} = 4 > 0$

$\Rightarrow$ at $x = 1$ we have minimum value of Z

and Minimum value of $Z = 2x^2 - 4x + 1$

$$= 2(1)^2 - 4(1) + 1$$

$$= 2 - 4 + 1 = -1$$

ex) find the Maximum value of the objective function $Z = 2X^2 - 3XY + 2Y + 10$ subject to the constraint $y = X$

حل نعوض Y باقتران objective
(subject)

$Z = 2X^2 - 3X^2 + 2X + 10$

$Z = -X^2 + 2X + 10$

$Z' = -2X + 2$

$Z' = 0$

$-2X + 2 = 0$

$2 = 2X$

$\boxed{X = 1}$

$Z'' = -2 < 0$

at $X = 1$ function has Maximum

التفاؤل هو الإيمان الذي يؤدي إلى الانجاز

من اقتران ال subject

$Y = X$

$X = 1$

$Y = 1$

$Z = 2X^2 - 3XY + 2Y + 10$

$= 2(1)^2 - 3(1)(1) + 2 + 10$

$= 2 - 3 + 2 + 10 = 11$

Max value of function objective $= 11$

ex) find the Maximum value of the objective function

$Z = -3X^2 + 6X + 2Y + 10$

Subject to the Constraint $Y - X^2 = 2$

الحل

① نخلي Y موضوع القانون (subject)

$Y = 2 + X^2$

② نعوض Y باقتران الـobjective

$Z = -3X^2 + 6X + 2(2 + X^2) + 10$

$Z = -3X^2 + 6X + 4 + 2X^2 + 10$

$Z = -X^2 + 6X + 14$

$Z' = -2X + 6$

$Z' = 0$

$-2X + 6 = 0$

$\frac{-2X}{-2} = \frac{-6}{-2}$

$\boxed{X = 3}$

$Z'' = -2 < 0$

$Z''\big|_{X=3} = -2 < 0$

at $X = 3$ function has Maximum

نعوض باقتران الـsubject $Y = 2 + X^2$

$Y = 2 + (3)^2 = 2 + 9 = 11$

نعوض باقتران الـobjective $(X = 3)$ $(Y = 11)$

$Z = -3(3)^2 + 6(3) + 2(11) + 10$

$= -27 + 18 + 22 + 10$

$= -9 + 10 + 22 = 23$

Max value of function objective $= 23$

ex) If the Profit function is $\pi = 40Q_1 - Q_1^2 + 2Q_1Q_2 + 20Q_2 - Q_2^2$
is subjected to the Constraint (قيد) $Q_1 + Q_2 = 15$
find the Maximum Profit.

① من اقتران الـ subject نجد Q_2 موضوع قانون

$$Q_2 = 15 - Q_1$$

② نعوض Q_2 في اقتران (objective)

$$\pi = 40Q_1 - Q_1^2 + 2Q_1(15 - Q_1) + 20(15 - Q_1) - (15 - Q_1)^2$$

$$= 40Q_1 - Q_1^2 + 30Q_1 - 2Q_1^2 + 300 - 20Q_1 - (225 - 30Q_1 + Q_1^2)$$

$$= 40Q_1 - Q_1^2 + 30Q_1 - 2Q_1^2 + 300 - 20Q_1 - 225 + 30Q_1 - Q_1^2$$

$$\pi = 80Q_1 - 4Q_1^2 + 75$$

مشتقة أولى ← $\frac{d\pi}{dQ_1} = 80 - 8Q_1$

$$\frac{d\pi}{dQ_1} = 0$$

$$80 - 8Q_1 = 0$$

$$80 = 8Q_1 \implies \boxed{Q_1 = 10}$$

مشتقة ثانية ← $\frac{d^2\pi}{dQ_1^2} = -8$

$$\left.\frac{d^2\pi}{dQ_1^2}\right|_{Q_1 = 10} = -8 < 0$$

$\implies$ we have Maximum value of π at $Q_1 = 10$

الآن نجد Q_2 من خلال التعويض بـ

$$Q_2 = 15 - Q_1$$

$$Q_2 = 15 - 10 = 5$$

for $\pi(Q_1, Q_2) = (10, 5)$ is a st. Point

$\implies$ Maximum Profit is $\pi = 40Q_1 - Q_1^2 + 2Q_1Q_2 + 20Q_2 - Q_2^2$

$$= 40(10) - (10)^2 + 2(10)(5) + 20(5) - (5)^2$$

$$= 400 - 100 + 100 + 100 - 25$$

$$= 475$$

Q) $f(x,y) = x^2 e^{xy} + 5xy$ find f_x, f_y ?

$$f_x = x^2 (y e^{xy}) + e^{xy}(2x) + 5y$$
$$= x^2 y e^{xy} + 2x e^{xy} + 5y$$

$$f_y = x^2 (x e^{xy}) + 5x$$
$$= x^3 e^{xy} + 5x$$

$x^2 e^{xy}$
مثلاً طالب f_y
ف x^2 تعتبر كثابتة

ex) If $z = x^3 y + y^3 x$ find z_x and z_y at the Point $(1,3)$.

$$z_x = 3x^2 y + y^3$$
$$z_x(1,3) = 3(1)^2(3) + (3)^3$$
$$= 9 + 27 = 36$$
$$z_y = x^3 + 3y^2 x$$
$$z_y(1,3) = (1)^3 + 3(3)^2(1)$$
$$= 1 + 27 = 28$$

أسئلة متنوعة

Q1) let $f(x,y) = \sqrt{xy^2}$ Then $\frac{\partial f}{\partial x}$ equals

Q2) let $f(x,y) = 4$ be an equation such that $f_x = 2x + 6y^2$ and $f_y = 3y^2 + 12xy$. then $\frac{dy}{dx}$ when $x = 0$ equals

① 2 ② -1 ③ -2 ④ $6y^2$ ⑤ $3y^2$

الحل

$$\frac{dy}{dx} = \frac{-f_x}{f_y}$$

when $x = 0$

$$= \frac{-2x - 6y^2}{3y^2 + 12xy} = \frac{-6y^2}{3y^2} = \boxed{-2}$$

Q3) The objective function $f(x,y) = 2x^2y + 9xy + 12x + 4$ subject to the Constraint $y = x$ has stationary Point at (a,a) Then a equals =

① 3 or 1 ② 0 or 2 ③ -2 or -1 ④ -3 or 4 ⑤ 5 only

Q4) If $f_x = y$ and $f_y = x$ then the Point $(0,0)$ is

Ⓐ a St. Point but we cannot determine its type

Ⓑ a saddle Point Ⓒ Not a stationary Point

Ⓓ a maximum Point Ⓔ a Minimum Point

Q5) $f(x,y) = (x^4)\sqrt{y} + e^{xy} + 3$ then f_{xy} =

Q6) A Firm Produces two goods G_1 and G_2 The Price are related To Quantities Q_1 and Q_2 according to the demand equations.

$P_1 = 1200$, $P = 1000$

If Total cost function is $TC = 2Q_1^2 + 2Q_1Q_2 + Q_2^2$

a) find the Profit function

b) find (Q_1, Q_2) that Maximizes the Profit function.

$f_{yx}(2,1)$ مثلاً

$y \to 2$
$x \to 1$

Q7) let $f(x,y) = x^2 + 3xy + y^6 = 4$ find $\frac{dy}{dx}$

Q8) $F(x,y) = 4e^{xy}$ find $f_{xy}(\ln 2, 1)$

$x \to \ln 2$
$y \to 1$

Q9) let $f(x,y) = 2e^{xy} + x^2 + y^2 = -3$ find $\frac{dy}{dx}$

Q10) $f(x,y) = x^2 y^3 - x^3 y^2 - xy + 5$
find $f_{yx}(1,1)$

Q11) find the Maximum value for $f(x,y) = 16x^2 - 2y^2 + 9y + 4x + 2xy$
subject to the Constraint $y = 4x$

Q12) find the Maximum value of $f(x,y) = x^2 + xy - y^2 + 30x$
subject to the Constraint $y = 3x$

Q13) If $f(x,y) = x^2 y^3 + x^3 y^2 + xy + 5$ then $f_{yx}(-1,1)$ is

Q14) If $f(x,y) = 3e^{xy} + x^3 - y^3 = -3$ then $\frac{dy}{dx}$

Q15) If $f(x,y) = x\sqrt{1+y^2}$ then $f_x =$

Q16) If $(x,y) = x^2 y^2 - 4\sqrt{x}\,y - y^3 + \ln xy$ then

a) $f_x =$

b) $f_{xy} =$